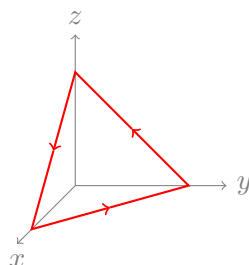


Calculus IV Homework 3

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due Friday, September 20, 2024

1. Let $\mathbf{F} = x^2 \mathbf{i} - y^2 \mathbf{k}$, and let C be the triangular path from $(1, 0, 0)$ to $(0, 1, 0)$ to $(0, 0, 1)$ back to $(1, 0, 0)$ shown in the diagram below.



Find the circulation of $\mathbf{F}$ around C .

2. Let $\mathbf{F} = \mathbf{i} + x^2 \mathbf{j}$, and let C be the boundary of the region $\{(x, y) : x^2 + y^2 \leq 1 \text{ and } x \geq 0\}$: the right half of the unit disk. Find the outward flux of $\mathbf{F}$ across C .
3. Determine whether these vector fields are gradient fields, and if they are, find a potential function for them.
- (a) $\mathbf{F} = 4x^2y \mathbf{i} + \frac{4}{3}(x^3 - y^3) \mathbf{j}$.
 - (b) $\mathbf{G} = z \cos(y + z) \mathbf{i} - xz \sin(y + z) \mathbf{j} - xz \sin(y + z) \mathbf{k}$.
 - (c) $\mathbf{H} = (x + y) \mathbf{i} + (x - z) \mathbf{j} - (y + z) \mathbf{k}$.
4. (a) Let $f(x, y, z) = xe^{y-z}$. Compute the gradient field $\mathbf{F} = \nabla f$.
- (b) Let C be the coiled spring parameterized by $\mathbf{r}(t) = (\cos t, \sin t, t/\pi)$, where $t \in [0, 4\pi]$. Compute the vector line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}$ is the gradient field from part (a).
5. Let $\mathbf{F}$ be an unknown, but conservative, vector field. Define paths
- C_1 , the line segment parametrized by $\mathbf{r}_1(t) = (2t - 1, 0)$, where $t \in [0, 1]$.
 - C_2 , the quarter-circle parametrized by $\mathbf{r}_2(t) = (\cos t, \sin t)$, where $t \in [0, \pi/2]$.
 - C_3 , the parabolic curve parametrized by $\mathbf{r}_3(t) = (t^2 - 1, -t)$, where $t \in [0, 1]$.
 - C_4 , the cubic curve parametrized by $\mathbf{r}_4(t) = (t^3 - t, t)$, where $t \in [-1, 1]$.
- If $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}_1 = 2$, $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}_2 = 3$, and $\int_{C_3} \mathbf{F} \cdot d\mathbf{r}_3 = 4$, find $\int_{C_4} \mathbf{F} \cdot d\mathbf{r}_4$.