Calculus IV Homework 5

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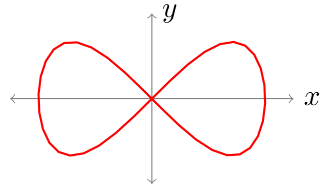
due Friday, October 18, 2024

1. The curve C shown below has parameterization $\mathbf{r}(t) = (\sin t, \sin t \cos t)$ where $t \in [0, 2\pi]$.



   Use Green's theorem to write $\int_C \mathbf{F} \cdot d\mathbf{r}$ as a double integral of the curl of $\mathbf{F}$. (Make sure to correctly find the bounds, and to correctly take the orientation of the curve into account.)

   Clarification: your answers should be written in terms of an arbitrary vector field $\mathbf{F}$ and its curl $\nabla \times \mathbf{F}$, since you are not given a formula for $\mathbf{F}$ in this problem.

2. Find parameterizations of the following surfaces:
   (a) The rectangle with corners $(1, 0, 0)$, $(1, 0, 1)$, $(0, 1, 0)$, and $(0, 1, 1)$.
   (b) The portion of the sphere $x^2 + y^2 + z^2 = 1$ with $x \geq 0$.
   (c) The portion of the cone with equation $x^2 + y^2 = z^2$ bounded by $1 \leq z \leq 3$.

3. The cylinder with equation $x^2 + y^2 = 1$ bounded by $0 \leq z \leq 1$ is parameterized by $\mathbf{r}(u, v) = (\cos u, \sin u, v)$, where $u \in [0, 2\pi]$ and $v \in [0, 1]$. Modify this parameterization in the following ways:
   (a) Rotate the cylinder to be centered around the y-axis instead: the result should be the cylinder with equation $x^2 + z^2 = 1$ bounded by $0 \leq y \leq 1$.
   (b) Shift the cylinder by 1 unit in the y-direction: the result should be the cylinder with equation $x^2 + (y - 1)^2 = 1$ bounded by $0 \leq z \leq 1$.
   (c) Make the cylinder 4 times wider and 2 times longer.

4. Use an integral to find the area of the surface parameterized by $\mathbf{r}(u, v) = (u \cos v, u \sin v, u^2)$, where $u \in [-1, 1]$ and $v \in [0, \pi]$.

5. Set up, but do not evaluate, each of the following surface area integrals.
   (a) The surface area of the portion of the surface $z = x^2 - y^2$ satisfying $0 \leq x \leq y \leq 1$. Use the formula for a surface given by $z = h(x, y)$.
   (b) The surface area of the portion of the surface $xyz = 1$ satisfying $1 \leq x \leq 2$ and $1 \leq y \leq 2$. Use the formula for a surface implicitly given by $f(x, y, z) = 0$.