Linear Programming Homework #6

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- 1. In the game of Live-Action Rock-Paper-Scissors, both players must acquire a physical object (Rock, Paper, or Scissors) with which to play the game. The rules are still the same: rock beats scissors, scissors beats paper, and paper beats rock. However, the players must take the value of their objects into account:
 - To play Rock, it's enough to find a rock lying on the ground (for free).
 - To play Paper, a player must spend \$2 to buy a ream of paper.
 - To play Scissors, a player must spend \$3 to buy cheap plastic scissors.

Even the most expensive purchase can be worth it, because the loser of the game must pay \$5 to the winner. (If there is a tie, neither player pays the other.)

- (a) Write down a linear program that finds the maximin strategy for this game: the mixed strategy that maximizes the expected net profit against the worst-case counter-strategy.
- (b) Is this a zero-sum game? Why or why not?
- 2. Find examples of networks with the following properties. For each example, describe the maximum s t flow(s) and the minimum s t cut(s).
 - (a) A network with a unique maximum s t flow, but multiple minimum s t cuts.
 - (b) A network with multiple maximum s t flows, but a unique minimum s t cut.
 - (c) A network with multiple maximum s t flows and multiple minimum s t cuts.
- 3. Find an augmenting path for the s t flow given in the diagram below. Then, use the path you found to augment the flow as much as possible.



4. The diagram below gives a residual graph for a network flow problem. (Solid arcs are "forward" arcs, dashed red arcs are "backward" arcs.)



- (a) Use the residual graph to reconstruct the original network, determining the arcs it has and their capacities.
- (b) Find the s t flow which produces this residual graph.
- (c) Find an s t cut whose capacity is equal to the value of the flow.
- 5. Six universities send a few of their students to a week-long program to learn more about linear programming. There is 1 student from the first university, 2 students from the second, 3 from the third, and so on, with the sixth university sending 6 students.

The students are divided to work on different projects. There are five projects that can accommodate up to 3 people, and one project that can accommodate any number; however, to foster inter-university collaboration, the students working on each project must all be from different universities.

Draw a network in which a maximum s - t flow can be used to determine how to assign the students to projects to satisfy these conditions. Do not solve.