

Graph Theory Homework 7

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1 Short answer

1. Let G be a connected graph with a plane embedding such that at every vertex, one face of length 5 and three faces of length 3 meet.

Determine the number of vertices, edges, and faces in G .

2. Let G be the interval graph whose vertices are the intervals

$[1, 6], [2, 4], [3, 14], [5, 13], [7, 8], [9, 11], [10, 12]$

with an edge between two vertices whenever they overlap.

- (a) Draw a diagram of G .
 - (b) What is the clique number of G ? Find a clique of that size.
 - (c) What is the independence number of G ? Find an independent set of that size.
3. Draw a graph with 10 vertices that has clique number 3 and independence number 4.

2 Proof

4. Determine the minimum number of edges in an $3n$ -vertex graph with an independence number of $2n$.

Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 5.

5. Prove that there is no 4-regular planar graph in which all faces have length 4.

You have already written a rough draft of the solution; now, write a final draft.