Math 482: Topics Covered in the Final Exam

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The final exam will be cumulative, but with added focus on the material covered between Friday, April 17$^{\text{th}}$ and Monday, May 4$^{\text{th}}$ (which we haven't had an exam on yet). In this document, I go through the new material and point out the things you should know from it.

1 Things you should know

Here are the definitions you should know:

- **Totally unimodular** matrices.
- **Integer programs**. (Formally, "integer linear programs" or "mixed integer linear programs".)
- Logical expressions in **conjunctive normal form** (an AND of many OR statements).
- Branch-and-bound terminology. We call each LP we solve a **node** and either **prune** it or **branch** from it.
- **Cutting planes** and specifically the **Gomory fractional cut**.
- The **restricted primal** in the primal-dual simplex method; tight constraints and **frozen variables/columns**.
- The **traveling salesman problem**; **tours** and **subtours**.
- The **approximation ratio** of an approximation algorithm.

You should know and understand the following results:

- That the primal-dual algorithm finds the optimal solution if/when it terminates.
- That nonzero variables in the optimal solution to one iteration of the restricted primal will remain present (or unfrozen) in the next iteration.
- That the fractional cut is valid for all integer points of a linear program.
- That both the Dantzig–Fulkerson–Johnson and the Miller–Tucker–Zemlin constraints for the TSP guarantee that the optimal solution to the integer program is a single connected tour.
- The 2-approximation guarantees of the algorithms for vertex cover. (The 2-approximation algorithm for TSP will not be on the exam.)

2 Things you should be able to do

- Write down integer program formulations for various problems (including logical expressions).
- Use either the branch-and-bound method or the cutting plane method (with fractional cuts) to solve an integer linear program.
- Use the primal-dual simplex method to solve a linear program. This includes:
  - The basic method: taking a dual feasible solution, writing down the restricted primal, solving it, finding the dual, and augmenting to get a better dual feasible solution.
  - Starting the method: using an upper bound on the primal variables to transform the problem into one that has a starting dual feasible solution, when the zero vector doesn't work.
  - Using the previous restricted primal solution for the restricted primal in the next iteration: by using the frozen columns approach, or the revised simplex method.