Math 482: Linear Programming	Mikhail Lavrov

Homework #6

Spring 2020 — *Due Friday, March 27*

1. You are given a set of points on a line at locations $a_1, a_2, \dots, a_n$. Write down linear programs to find a point x on that line that minimizes
   (a) The maximum distance from x to any of the points.
   (b) The sum of the distances from x to all of the points.

   (Do not solve either of the linear programs.)

2. On the exam, a zero-sum game with the payoff matrix below was discussed:

	Bob : a	Bob : b	Bob : c
Alice : A	-2	3	-4
Alice : B	5	-6	7

   The optimal strategy for Bob involves choosing option a with probability 0, not because a is dominated by b or c, but because a is dominated by a mixed strategy.

   Find the range of values of p such that Bob playing option b with probability p and option c with probability $1-p$ dominates option a.

3. Alice and Bob each have two coins: a nickel (5 cents) and a dime (10 cents). They simultaneously put a coin down on the table. If the coins are equal in value, Alice wins Bob's coin; if Alice's coin is more valuable, Bob wins Alice's coin; if Bob's coin is more valuable, nothing happens.

   Determine optimal strategies for Alice and Bob.

4. Use Fourier–Motzkin elimination to find a point (x, y, z) satisfying

$$\begin{aligned} x \qquad - z &\le 1 \\ x + y + z &\le 1 \\ -x + y + 3z &\le 1 \\ -3x - 5y - z &\le -1 \end{aligned}$$

5. *(Only 4-credit students need to do this problem.)*

   Use Farkas's lemma to prove LP duality in the following form: if the linear program (**P**) below *cannot achieve* an objective value of at least z^*, and the dual program (**D**) is feasible, then the dual linear program (**D**) has a feasible solution $\mathbf{u}$ with objective value less than z^*.

$$(\mathbf{P}) \begin{cases} \underset{\mathbf{x} \in \mathbb{R}^n}{\text{maximize}} & \mathbf{c}^\mathsf{T}\mathbf{x} \\ \text{subject to} & A\mathbf{x} \le \mathbf{b} \end{cases} \qquad (\mathbf{D}) \begin{cases} \underset{\mathbf{u} \in \mathbb{R}^m}{\text{minimize}} & \mathbf{u}^\mathsf{T}\mathbf{b} \\ \text{subject to} & \mathbf{u}^\mathsf{T}A = \mathbf{c}^\mathsf{T} \\ & \mathbf{u} \ge \mathbf{0} \end{cases}$$