Math 482: Linear Programming	Mikhail Lavrov
Homework $\#8$	
Spring 2020	Due Friday, April 10

1. Given the network below, with label x/y denoting a flow of x and a total capacity of y along an edge, draw the residual graph, and use it to list all possible augmenting paths.



2. The diagram below gives a residual graph for a network. (Black edges are "forward" edges, red dashed edges are "backward" edges.)



- (a) Determine the edges of the original network, and their capacities.
- (b) Find the flow which produces this residual graph.
- (c) Find a cut with the same capacity as the value of this flow.
- 3. Consider the following matching (that is, $M = \{(a, d), (b, e)\}$) in a bipartite graph:



First, convert this matching into a feasible flow in a network. Then, find an augmenting path in that network, and use it to improve the matching to a larger one.

4. Suppose that we want to find a feasible circulation in the network below with flows on each edge in the specified lower and upper bounds.



Find a feasible circulation in this network by writing down an equivalent maximum-flow problem, finding the maximum flow in that network, and converting it back to an equivalent feasible circulation.

5. (Only 4-credit students need to do this problem.)

Write down a linear program for a general feasible circulation problem. (There is no objective function, so make the objective function just "maximize 0".)

Then, take the dual of this linear program.