Math 482: Topics Covered in Exam 2

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The second exam will, broadly speaking, cover all the material covered in class from Friday, February 21^{st} to Monday, March 9^{th}: linear programming duality.

The exam will not be cumulative in the sense that I won't ask problems that are about earlier topics. But you'll need to use a lot of the earlier ideas to solve problems about the new topics. In particular, you'd better be comfortable with the simplex method.

Below I try to summarize the important things we've covered that will be on the exam. It's possible that I've missed a few topics, and it's hard to summarize four weeks of content in two pages anyway; if you're not sure about something, ask me.

1 Things you should know

Here are the definitions you should know:

- The **dual** of a linear program. Associated terminology for the dual (e.g. **primal feasible** and **dual feasible** solutions).
- **Complementary slackness** (as well as **slack** and **tight** constraints).
- **Primal feasible** and **dual feasible** tableaux.
- **Zero-sum games**, **payoff matrices**, and **mixed strategies**. I didn't actually get around to saying the words **mixed strategy Nash equilibrium** in class, but it's the term for the pair of optimal mixed strategies we find for a zero-sum game.
- **Dominated strategies**, **dominant strategies**, and **saddle points** in a zero-sum game.

You should know and understand the following results about linear programming duality:

- Weak duality (the inequality between any primal feasible and dual feasible solutions).
- Strong duality (when a primal optimal and dual optimal solution both exist, their objective values are equal)
- The possible cases of when the primal and the dual are feasible/infeasible and bounded/unbounded.
- Complementary slackness.
- Sensitivity analysis: changes in the optimal solution when bounds on inequalities or objective coefficients are changed.

2 Things you should be able to do

Skills related to the dual of a linear program:

- Write down the dual of a linear program, keeping in mind the possible cases for constraints ($\leq$ or $=$ or $\geq$), objective function (min or max), and variables (nonnegative or unconstrained).
- Given the primal or dual optimal solution to a linear program, use complementary slackness to compute the other optimal solution.
- Given an optimal tableau for a linear program, find the corresponding optimal dual solution.

Skills related to the dual simplex method:

- For linear programs with inequality constraints in which the objective function is "minimize $\mathbf{c}^\mathsf{T}\mathbf{x}$" for some $\mathbf{c} \geq \mathbf{0}$, use the dual simplex method to solve the linear program.
- For general linear programs, use the dual simplex method as the first phase of a two-phase simplex method to find an initial feasible solution.

  (Don't forget that for linear programs with constraints $A\mathbf{x} = \mathbf{b}$, you should row-reduce to find a basic solution first.)
- Use the dual simplex method to find a feasible solution to a system of linear inequalities and equations.
- Use the dual simplex method to add an inequality constraint to an existing linear program, and re-optimize.

Sensitivity analysis:

- Use the primal and dual optimal solutions to predict how the optimal objective value of a linear program will change if the cost vector $\mathbf{c}$ or the RHS of the constraints $\mathbf{b}$ is changed slightly.
- Determine whether your prediction is a lower bound or an upper bound in general.
- Give a range (for each component of $\mathbf{b}$ or $\mathbf{c}$) within which you know that this prediction is correct.

Zero-sum games:

- Given the description of a two-player zero-sum game, write down a payoff matrix for the game.
- Given the payoff matrix, identify saddle points and dominated strategies (if any exist).
- Given the payoff matrix and some pure or mixed strategies for the two players, determine the expected payoff.
- Given the payoff matrix, write down the linear programs that find the optimal mixed strategy for each player.