Math 484: Nonlinear Programming	Mikhail Lavrov

Homework #1

Spring 2019 — *Due Friday, January 25*

1. On the first day of class, we discussed optimization problems in the standard form

$$\begin{aligned} \underset{x_1,x_2,\ldots,x_n\in\mathbb{R}}{\text{minimize}} \quad & f(x_1,x_2,\ldots,x_n) \\ \text{subject to} \quad & g_1(x_1,x_2,\ldots,x_n) \le 0, \\ & \ldots, \\ & g_m(x_1,x_2,\ldots,x_n) \le 0. \end{aligned}$$

More general problems can be put in this form. Suppose you are given the *maximization* problem:

$$\begin{aligned} \underset{x,y\in\mathbb{R}}{\text{maximize}} \quad & x+2y \\ \text{subject to} \quad & x^2+y^2=1. \end{aligned}$$

Show how to write this problem in the standard form above. (I'm not asking you to solve the problem.)

2. Find the local and global minimizers of the following functions:
   (a) $f(x) = x^3 + x + 1$.
   (b) $f(x) = e^x - 2x$.
   (c) $f(x) = \dfrac{x}{x^2+1}$.
   (Hint: the quantities $x^2 - 2x + 1 = (x-1)^2$ and $x^2 + 2x + 1 = (x+1)^2$ cannot be negative!)

3. Compute the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 4 & -6 \\ 3 & -5 \end{bmatrix}$.

4. For two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, the expression $\|t\mathbf{x} + (1-t)\mathbf{y}\|^2$ is a quadratic function of t.
   Simplify it to the form $at^2 + bt + c$, where a, b, c are in terms of $\mathbf{x}$ and $\mathbf{y}$.

5. *(Only 4-credit students need to do this problem.)*
   In this problem, we consider the function $f(x,y) = (y - x^2)(y - 2x^2)$, and its critical point at $(0,0)$.
   (a) Let $\phi(t)$ be the restriction of $f(x,y)$ to a line in the direction (u,v) from $(0,0)$: $\phi(t) = f(tu,tv)$. Show that 0 is always a local minimizer of $\phi(t)$, no matter which direction (u,v) we pick.
   (b) Show that $(0,0)$ is *not* a local minimizer of $f(x,y)$, despite it being "a local minimizer along every line through $(0,0)$".

General instructions for writing up homework:

- *If you're taking the class for 4 credits (as opposed to the default of 3), write this on your assignment so that it can be graded appropriately.*
- *When writing up solutions, if you use a result from your textbook, say the result you're using (by name, or theorem number, or whatever) and why it applies. E.g., "So $f''(5) = 1$. By the second derivative test, since $f''(5) > 0$, the critical point $x = 5$ is a strict local minimizer."*
- *You don't need to show your work for routine computations, but if you get those wrong without showing your work, you'll miss the opportunity for partial credit.*
- *Write proofs in complete sentences.*