Math 484: Nonlinear Programming | Mikhail Lavrov

Homework #10

Spring 2019 | *Due Monday, April 29*

1. If we use the method of steepest descent to minimize $f(x, y) = x^2 - xy + y^2$ starting from $(x_0, y_0) = (1, 2)$, compute the general form of the k^{th} iteration (x_k, y_k).

   (You will get slightly different expressions for odd k and for even k. You might want to begin by finding the first few points to see the pattern.)

2. Suppose that we want to use a descent method to minimize $f(x, y, z) = x^4 + y^2 + z^2 - 10xz$ starting from the point $\mathbf{x}^{(0)} = (1, 1, 1)$.

   Find a value of μ for which $-(Hf(\mathbf{x}^{(0)}) + \mu I)^{-1} \nabla f(\mathbf{x}^{(0)})$ will be a descent direction.

3. Suppose that we want to use a descent method to minimize $f(x, y) = x^3 + y^3$ starting from the point $\mathbf{x}^{(0)} = (1, 2)$ in the direction $\mathbf{p}^{(0)} = (1, -1)$.

   Find the range of the values t_0 such that going from $\mathbf{x}^{(0)}$ to

   $$\mathbf{x}^{(1)} = \mathbf{x}^{(0)} + t_0 \mathbf{p}^{(0)}$$

   will satisfy the criteria of Wolfe's theorem, with constants $\alpha = \frac{1}{4}$ and $\beta = \frac{1}{2}$.

4. For the matrix

   $$A = \begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix}$$

   find the "rank-one update" matrix U such that

   $$(A + U) \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

   while

   $$(A + U) \begin{bmatrix} 1 \\ -1 \end{bmatrix} = A \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$