Math 484: Nonlinear Programming	Mikhail Lavrov
Homework #2	
Spring 2019	*Due Friday, February 1*

1. Using any method you like, classify the matrices below as positive (semi)definite, negative (semi)definite, or indefinite.

$$\text{(a)} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(b)} \begin{bmatrix} 2 & 1 & 4 \\ 1 & 3 & 1 \\ 4 & 1 & -1 \end{bmatrix} \quad \text{(c)} \begin{bmatrix} 1 & -2 & 0 \\ -2 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

2. Find the 3×3 symmetric matrix A whose associated quadratic form $\mathbf{x}^\mathsf{T} A \mathbf{x}$ is $(x_1 + 2x_2 - x_3)^2$.

   Without computing any determinants or eigenvalues, how can you quickly tell that this quadratic form is positive semidefinite, but not positive definite?

3. Find the critical points of the function

$$f(x, y) = x^2 + y^3 - 3xy.$$

   Use the Hessian matrix of f to classify them as local minimizers, local maximizers, neither, or "can't tell based on the Hessian matrix only".

4. Classify the functions below as coercive or not coercive.

   (a) $f(x, y) = \dfrac{x^4}{x^2 + 1} + |y|$.

   (b) $g(x, y) = e^x + 2y^2$.

   (c) $h(x, y) = (x - y)^2$.

5. *(Only 4-credit students need to do this problem.)*

   (a) Suppose a symmetric $n \times n$ matrix A has a factorization $A = B^\mathsf{T} B$. Show that A is positive semidefinite.

   (b) Suppose that A is positive semidefinite. Find a factorization $A = B^\mathsf{T} B$.

   (Hint: start with the spectral theorem.)