Math 484: Nonlinear Programming	Mikhail Lavrov
Homework #5	
Spring 2019	*Due Friday, March 1*

1. (a) Find a polynomial $y = P(x)$ of degree at most 3 that passes through the points

$$\{(0,5),(1,1),(2,-1),(3,5)\}.$$

   (b) Describe all polynomials of degree at most 4 that pass through the points

$$\{(0,5),(1,1),(2,-1),(3,5)\}.$$

2. Given the vectors

$$\mathbf{a} = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 6 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

find the linear combination of **a** and **b** which is closest to **c**.

3. Find the distance between the origin $(0,0,0,0)$ and the closest point (x_1, x_2, x_3, x_4) satisfying the equations

$$\begin{aligned} x_2 + x_3 + 2x_4 &= 7, \\ 2x_1 - x_2 + x_3 + x_4 &= -4. \end{aligned}$$

4. Use the Gram–Schmidt process to find an orthonormal basis for the subspace of $\mathbb{R}^3$ spanned by

$$\mathbf{a}^{(1)} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{a}^{(2)} = \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}, \quad \mathbf{a}^{(3)} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{a}^{(4)} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$