Math 484: Nonlinear Programming | Mikhail Lavrov

Homework #7

Spring 2019 | *Due Friday, March 29*

1. For the (kind of silly) convex program

$$(P) \qquad \begin{cases} \underset{x \in \mathbb{R}}{\text{minimize}} & x^2 \\ \text{subject to} & x \le 0, \\ & -x \le 0 \end{cases}$$

find the value function $MP(z_1, z_2)$ explicitly in terms of z_1 and z_2.

2. Use the gradient KKT theorem to solve

$$\begin{cases} \underset{x,y \in \mathbb{R}}{\text{minimize}} & x - y \\ \text{subject to} & x^2 + y^2 \le 1, \\ & 2x - y \le -1. \end{cases}$$

Be sure to check a condition that guarantees that we get an optimal solution.

3. Consider the convex program

$$(P_c) \qquad \begin{cases} \underset{x,y \in \mathbb{R}}{\text{minimize}} & x + y \\ \text{subject to} & y^2 - x \le 0, \\ & x - y \le c. \end{cases}$$

For which values of c is P_c superconsistent?

Show that for *all* such values of c, the optimal solution (x^*, y^*) of P_c satisfies $x^* = (y^*)^2$.

4. Use the gradient KKT theorem to *fail to* solve

$$\begin{cases} \underset{x,y \in \mathbb{R}}{\text{minimize}} & x + 2y \\ \text{subject to} & x^2 + y^2 \le 1, \\ & 3x + 4y \le -5. \end{cases}$$

Why is the solution (x, y) to the gradient conditions definitely not an optimal solution of this convex program, and which requirements of the theorem are violated?