Math 484: Nonlinear Programming		Mikhail Lavrov
	Homework #9	
Spring 2019		*Due Friday, April 19*

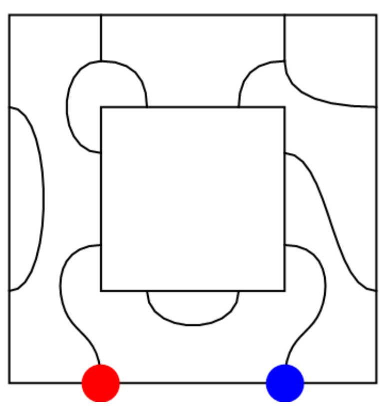
1. Use the penalty function method with the Courant–Beltrami penalty term to solve

$$\begin{aligned} \underset{x,y\in\mathbb{R}}{\text{minimize}} \quad & x^4 + y^2 \\ \text{subject to} \quad & x^2 + y^2 = 5. \end{aligned}$$

2. Show that the following functions are coercive:
   (a) $f(x, y) = x^2 + y^6 + 3xy - y^3$.
   (b) $f(x, y) = 3x^2 - 6xy + 4y^2$.
   (c) $f(x, y) = x^4 + y^2 + \frac{e^y}{x^2+1}$.
3. Do three iterations of Newton's method for solving $x^3 + 2x + 2 = 0$, starting from $x_0 = 0$.
4. Do two iterations of the secant method for solving $x^3 + 2x + 2 = 0$, starting from $x_0 = 0$ and $x_1 = 1$.

There was a lot of extra space on the bottom of this page, so just for fun, here is a puzzle appropriate to beginning the study of iterative methods.

This is an iterative fractal maze. The missing square in the middle of the maze should be replaced by a smaller copy of the maze; the missing square in the middle of that copy should be replaced by an even smaller copy of the maze; and so on.



Once that is done, your task (not worth any points for your homework grade, unfortunately) is to get from the red point to the blue point by following the curved lines: both in the maze and in its scaled copies.

(This fractal maze is not my creation; it can be found originally on http://www.mathpuzzle.com/23Dec2010.html.)